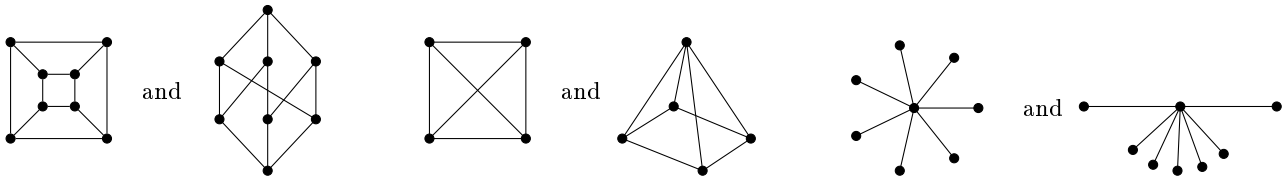
M314 REVIEW EXERCISES 29.03.17

You're encouraged to discuss these problems with other students in the class.

Dictionary:
A *walk* in a graph is a sequence of vertices such that a vertex can only follow another vertex it's adjacent to.
A graph is *connected* if there exists a walk from any vertex to any other vertex.
A *closed walk* is a walk that starts and ends at the same vertex.
A cicle/circuit is a closed walk where no edge is traversed twice. A graph that contains no cycles is *acyclic*.
A graph is a *tree* if it is connected an contains no cycles.

1. Let T be a graph with n vertices. Which one of these is NOT equivalent to saying that T is a tree?
   a) T is connected and acyclic.
   b) T has $n-1$ edges.
   c) For any two distinct vertices v, w in T, there is a unique path from v to w.
2. Prove that any tree with at least 2 vertices is a bipartite graph

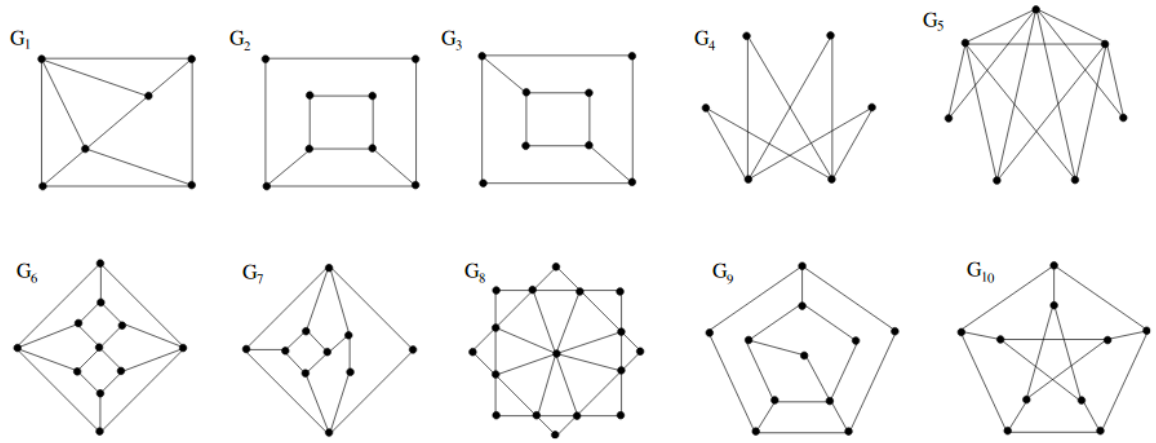
3. Which of these pairs of graphs are isomorphic?



4. The table lists the number t_n of non-isomorphic trees on n vertices. Find those trees.

n	1	2	3	4	5	6
t_n	1	1	1	2	3	6

5. Which of these graphs have an Eulerian circuit? Can you see a pattern?



Hint: compare G_4 and G_5.

6. Prove that if a graph is not a cycle and has a Hamiltonian circuit, then at least two vertices have degree ≥ 3.