M314 REVIEW EXERCISES 29.03.17

You're encouraged to discuss these problems with other students in the class.

Dictionary:
A graph $G = (V, E)$ is a data structure that consists of a set of vertices/nodes V and a relation E (edges) on this set.

Two vertices can have an edge between them or not. If they do, we say they are *adjacent*.

The *degree* of a vertex is the number of edges coming out of that vertex. A loop will count twice, since both endpoints are at the same vertex.

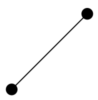
A *clique* is a graph where any two vertices are adjacent.

A graph $G = (V, E)$ is a *simple* if there are no edges from a vertex to itself ("loops") and between any two vertices there is at most one edge.

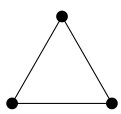
A graph $G = (V, E)$ is called *bipartite* if there exists a bipartition of the set of vertices into two sets A and B, such that no two vertices in A are adjacent and no two verices in B are adjacent.

1. In the following graphs, identify which ones are cliques, bipartite, simple? What is the degree of each vertex? How many edges does each of these graphs have?

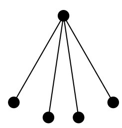
a)



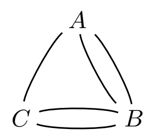
b)



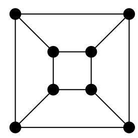
c)



d)



e)



2. The *handshake lemma* says that the sum of degrees of the vertices in the graph is twice the number of edges in the graph. Use that fact to answer the following question:

   Is it possible to connect 7 computers into a network by laying cables between (distinct) pairs of them so that every computer is connected to exactly three other computers?

3. What are the adjacency matrices of the graphs in question 1? You may have to index the vertices first. For example, in the already indexed graph d), we have:

d)

	A	B	C
A	0	2	1
B	2	0	2
C	1	2	0

4. Suppose we have a bipartite graph $G = (V, E)$, with *bipartition* A, B. A *perfect matching* in this graph is a bijection between A and B where a vertex can only be paired with another vertex if there is an edge between them. Of course, since this is a bijection we can only do it if A and B contain the same number of vertices. Can you come up with a graph like that where A, B do contain the same number of vertices, every vertex has at least degree 1 but a perfect matching is still impossible?