M314 REVIEW EXERCISES 08.02.17

You're encouraged to discuss these problems with other students in the class.

Dictionary:

An equation of the form

$$a_k = Aa_{k-1} + Ba_{k-2},$$

where $A,\ B$ are any reals (not depending on k), is called a *linear, homogeneous, recurrence relation with constant coefficients, of degree 2*. We'll call it a "degree-2 LHRR."
Characteristic equation (of an LHRR): a recurrence relation of the form

$$a_k = Aa_{k-1} + Ba_{k-2}$$

has a *characteristic equation*

$$t^2 = At + B.$$

We can find the closed formula of these sequences by finding the roots of this equation.

1. Each of 26 people is given a set of 9 balls numbered from 1 to 9 as pictured. Each of them can choose at most three of them. Show that there must be two people with the same sum of numbers on the balls they chose.

2. For the Fibonacci Sequence $\{a_n \mid n \in \mathbb{N}\}$, defined by $a_0 = 0,\ a_1 = 1,\ a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$, show that:

$$\forall n \in N,\ a_0 + a_1 + \cdots + a_n = a_{n+2} - 1$$

   We need to do this by induction. What is the base case? What is the inductive step?

3. Find the closed formula of:
   $a_1 = 1$
   $a_n = 2a_{n-1} + 1,\ \forall n \geq 2$

4. We want to find the closed formula of the Fibonacci Sequence (starting at $a_0 = 0,\ a_1 = 1$), by using the characteristic equation. Do the following:
   - Write down the characteristic equation of the Fibonacci Sequence.
   - Find the roots $r,\ s$ of this equation.
   - Find constants $C,\ D$ such that:

   $$a_0 = C + D$$

   $$a_1 = Cs + Dr$$

     Where $r,\ s$ are the roots of $t^2 = At + B$.
   - The closed formula is:

   $$a_k = Cr^k + Ds^k$$

   Verify your result for the first few terms.