M314 REVIEW EXERCISES 01.02.17

You're encouraged to discuss these problems with other students in the class.

Dictionary:

1. Use induction to prove that for all natural numbers $n \geq 1$:

$$1 + 2 + 3 + \cdots + (n-1) + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

   Remember that you need to:
   - Demonstrate that the base case is true.
   - Prove the inductive step.

2. Use induction to prove that for all natural numbers $r \neq 1$, and natural numbers $n \geq$:

$$r^0 + r^1 + r^2 + \cdots + r^n = \sum_{k=0}^{n} r^k = \frac{r^{n+1} - 1}{r - 1}$$

   What is your base case and what are the inducton steps? (i.e. what are we counting?)

3. Define a sequence $a_1,\ a_2,\ a_3,\ \ldots$ as follows: $a_1 = 1$, $a_2 = 1$ $a_3 = 1$, and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for all integers $n \geq 4$. Use strong mathematical induction to prove that $a_n \leq 2^n$ for all integers $n \geq 1$.

   How many terms are you going to need to show that the predicate is true for the next one?