M314 REVIEW EXERCISES 18.01.17

You're encouraged to discuss these problems with other students in the class.

Sets:
A set S with elements -1, 0, 1 can be written as: $S = \{-1, 0, 1\}$
$0 \in S$ "0 belongs to S"
$2 \notin S$ "2 does not belong to S"
$\mathbb{N} \subseteq \mathbb{Z}$ "The set of natural numbers is a subset of the set of integers."

Predicates:
$P(x, y)$ means the statement $P(\_, \_)$ evaluated on x and y.

Quantifiers:
$\forall$ reads as "for all". $\forall x \in S, P(x)$ reads as "For all x in the set S, P is true for x."
$\exists$ reads as "there exists". $\exists x \in S, P(x)$ reads as "There exists x in se S such that P is true for x."

1. What do you think the variation $\mathbb{N} \subset \mathbb{Z}$ might mean?
2. For each of these predicates on the set of integers, identify the truth set.
   - x is divisible by 2.
   - x is both prime and odd.
   - x is both prime and negative.
   - x is a real number.
3. A predicate on a Cartesian product of sets is called a *relation*. Explain why that could be.

   The set a predicate operates on is called *domain*. Why?
4. Are these statements true? What are their negations?
   - $\forall x \in \mathbb{R},\ x^2 \geq 0$
   - $\forall x \in \mathbb{R},\ x^2 \neq 1$
   - $\forall x \in \mathbb{R},\ (x > 2 \ \rightarrow\ x^2 \geq 4)$
5. As an exercise, write out these statements in English:

$$\neg(\forall x \in S,\ P(x)) \equiv \exists x \in S,\ \neg P(x)$$

$$\neg(\exists x \in S,\ P(x)) \equiv \forall x \in S,\ \neg P(x)$$

$$\neg(\forall x \in S,\ \exists y \in T, P(x, y)) \equiv \exists x \in S,\ \forall y \in T,\ \neg P(x, y)$$

6. For the instance of Tarski's World displayed on the screen, determine whether these are true. Then write them down formally and write down the negation:

   a. For all squares x, there is a circle y such that x and y have the same color.

   b. There is a triangle x such that for all squares y, x is to the right of y.

   c. For all circles x, x is above f.

   d. For all circles x, there is a square y such that x and y are the same color.