

Lecture 24 Worksheet

Math 20 Fall 2014, Dartmouth College

I. Which of the following matrices are transition matrices for regular Markov chains?

a.

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

b.

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0 \end{bmatrix}$$

c.

$$P = \begin{bmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{bmatrix}$$

d.

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

e.

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

II. A cat and a mouse are in a two-room apartment. At each time step, the cat will stay in the same room with probability 0.2 and go to the other room with probability 0.8. If the mouse is in room 1, it will stay with probability 0.7 and go to room 2 with probability 0.3. If the mouse is in room 2, it will stay with probability 0.4 and go to room 1 otherwise.

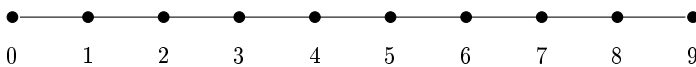
- Find stationary distributions of the cat and mouse Markov chains.
- Set up a joint Markov chain with four different states (how?). What is the expected time until they are in the same room?

III. Consider the Markov chain with transition Matrix:

$$P = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$$

- a. Show that this is a regular Markov chain.
- b. The process is started in state 1, find the probability that it is in state 3 after two steps.
- c. Find the stationary distribution.

IV. Is a simple random walk on the path pictured below an ergodic Markov chain? Is it regular?



V. Toss a fair die repeatedly. Let S_n denote the sum of the outcomes after n tosses. Let P_n be the proportion of the first n values S_n that are divisible by 7. It converges to a limit. Find this limit, by setting this process up as a 7-state Markov chain.

VI. Prove that in an r -state ergodic chain it's possible to go from any state to any other state in at most $r - 1$ steps.

VII. Consider a Markov chain with the following transition matrix, for some a, b :

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

- a. Under what conditions is P absorbing?
- b. Under what conditions is P ergodic but not regular?
- c. Under what conditions is P regular?