#### Lecture 16: Puzzles

## Math 20 Fall 2014, Dartmouth College

# 1 Roulette

Roulette in Vegas works as follows: there are numbers from 1 to 36, colored either black or red, and two extra slots numbered 0 or 00. You can bet on any slot you want, or some of the properties of the numbers. If you bet on a slot, you get a return of 36 times what you bet if you win. In any other case, you get a return as if the slots 0 and 00 were not there - in any case, these two slots are what makes money for the casino.

Alice is in Vegas on her 21st birthday. She has \$105 and wants to play roulette. She's going to make 105 \$1 bets on slot 21. What is the probability that she comes out ahead?



How do you resolve your answer with the law of large numbers?



## 2 Russian Fairy Tale

In rural Russia in the old days, children used to play the following game to divine an answer to a yes or no question. Someone would take three pieces of ribbon, which together have 6 ends, and would hold the ribbons so that the person asking the question can only see the ends. Then the asker would tie three pairs of ends together. If the result is a single loop, the answer is yes. What is the probability of a single loop?

## 3 Choosing Life

You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 red balls, 50 blue balls and 2 empty bowls. He then says, "Divide these 100 balls into these 2 bowls. You can divide them any way you like as long as you use all the balls. Then I will blindfold you and mix the bowls around. You then can choose one bowl and remove ONE ball. If the ball is blue you will live, but if the ball is red... you will die."

How do you divide the balls up so that you have the greatest probability of choosing a blue ball?

#### 4 Plane Seats

100 people get on a plane with 100 seats. Unfortunately, the first passenger through boarding is a mathematician trying to solve some very absorbing problem, and he manages to lose his boarding pass on the way from the gate to the plane. Once on the plane, he has no idea which is his seat. He picks a seat uniformly at random. Now, every passenger that comes in will pick their seat if it's empty, and if it's not, will pick an empty seat uniformly at random. What is the probability that the last person on board sits in their own seat?

#### 5 Abducted by Aliens

A fleet of alien starships arrives on Earth with the intention of abducting a group of humans. They pick a household with 5 men and 8 women. They are going to beam them all up at random. However, these particular aliens believe in strict separation of men and women, so as they start beaming the humans up, as soon as they beam up a human of the opposite gender than the previous one, they beam him or her back down and the next starship starts beaming the remaining humans up in the same manner. What is the probability that the last human beamed up is a woman?