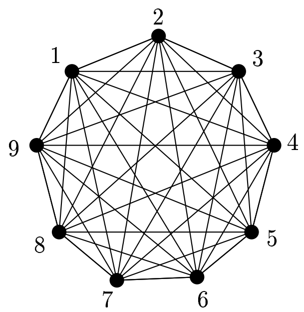
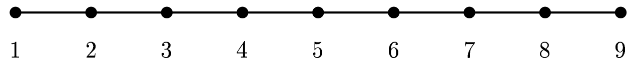
MATH 20 FALL 14 ASSIGNMENT 7, DUE FRIDAY 10/31

You're encouraged to discuss these problems with other students in the class. Hand in the solutions on paper at the beginning of class on Friday, 11/14. Also, be careful - the book uses old names for some of these things, for example "hitting time" is "mean first passage time" and "stationary distribution" is "fixed row vector" there.

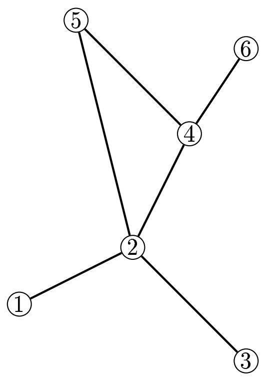
1 Random walks practice

I. Find hitting times h_{13}, h_{17} and h_{19} for each of these two graphs:

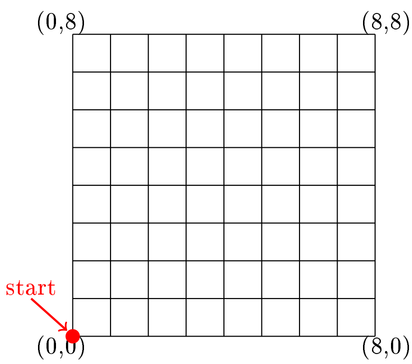




II. What is the stationary distribution of a simple random walk on this graph?



III. A token starts at point (0,0) on the grid pictured below. At each step, it either takes a step of length 1 to the right, or a step of length 1 up, each with probability 1/2. What is the most likely point it will be at after 8 steps? What are all the possible points it could be after 8 steps? What is the probability distribution?



2 Transition probabilities

I. In a famous psychology experiment, Asch (1951) examined the degree to which people conform to group pressure. The details of the experiment are online if you're curious. Eventually, the subjects were classified into four categories: permanent nonconformist, temporary nonconformist, temporary conformist, permanent conformist. Each subject's current category was said to be modeled as a Markov chain, with the two "permanent" categories as absorbing states and transition probabilities as follows:

$$P = \begin{array}{c} PNC \\ TNC \\ TC \\ PC \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.06 & 0.63 & 0.31 & 0 \\ 0 & 0.46 & 0.49 & 0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the fundamental matrix, find the times to absorption and absorption probabilities in this chain.

II. Check if the Weather in the Land of Oz is a reversible Markov chain.

$$P = \begin{array}{c} R \\ N \\ S \end{array} \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

III. Show that any ergodic Markov chain with a symmetric transition matrix (i.e. if $P_{ij} = P_{ji}$) is reversible.

IV. There are two urns with a total of $2N$ balls. Initially, the first urn has N red balls and the second urn has N blue balls. At each stage, we pick one ball uniformly at random from each urn and interchange them. Let X_n be the number of blue balls in the first urn at time n. This is a Markov chain on state space $\{0, 1, 2, 3, \ldots, N\}$.

a. Find the transition probabilities of this chain.
b. Show that $(s_0, s_1, \ldots, s_N)$, where:

$$s_i = \frac{\binom{N}{i}\binom{N}{N-i}}{\binom{2N}{N}}$$

   is the stationary distribution by verifying the reversibility condition.